Name ____

Graph Theory Project

Congratulations! You are now the CEO of Cool Stuff Inc. and must travel to your regional production facilities to assess the company's current performance. Being the new CEO, you wouldn't dare squander funds. Thus, you will find efficient travel plans to help save the company money. While you sit in your Seattle penthouse, you begin to plan your trip. Here are the necessary travel plans:

There are 6 main production facilities for Cool Stuff Inc., which all happen to be in a city with airport access. The cities are:

Seattle, WA

Dallas, TX

Atlanta, GA

Newark, NJ

Buffalo, NY

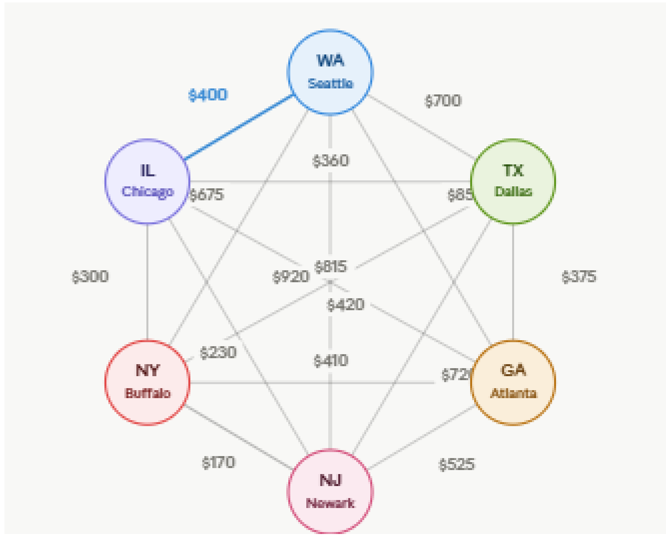
Chicago, IL

You must travel to all six of these facilities at least once. The airline ticket price to travel between these cities is provided in the following chart:

1. Draw a complete, weighted graph that shows all possible travel options

, assuming you visit each city only once and must return home. You can draw

on the back of this page or on a separate sheet of paper. Make your graph



2. Use the nearest neighbor method to find a route that is approximately the most optimal route to take. How much does this route cost? Why can you be sure that this is not the most expensive route?

Steps, starting at WA:

Step	Current City	Cheapest Unvisited Option	Cost
1	WA	IL = 400	400
2	IL	TX = 360	360
3	TX	GA = 375	375
4	GA	NY = 410	410
5	NY	NJ = 170	170
6	NJ	Return to WA = 920	920

Route: WA → IL → TX → GA → NY → NJ → WA

Total Cost = 400 + 360 + 375 + 410 + 170 + 920

= $2,635

The nearest neighbor method always picks the *minimum* available edge at each step, so by definition, it avoids the most expensive choices along the way. It greedily minimizes cost at each stop, making it impossible to be the worst possible route.

3. Find two alternate routes for visiting each city when starting and ending in Seattle. These do not need to be optimal routes. How much does each of these new routes cost? Did you find a cheaper alternative to your previous travel

plans?

Route A: WA → TX → GA → NJ → NY → IL → WA

700 + 375 + 525 + 170 + 300 + 400

= $2,470

Route B: WA → NY → NJ → GA → TX → IL → WA

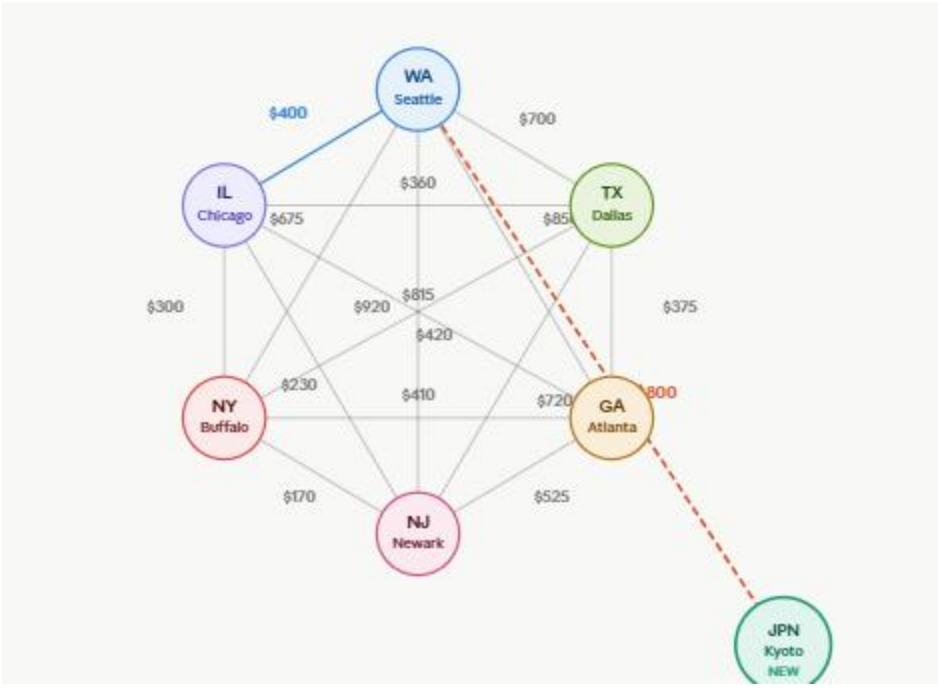
675 + 170 + 525 + 375 + 360 + 400

= $2,505

Yes! Route A ($2,470) is cheaper than the nearest neighbor route ($2,635). This shows the nearest neighbor method gives a good answer, but not always the best one.

4. Congratulations! After you have visited all the facilities, Cool Stuff Inc. has just opened a new international facility in Kyoto, Japan. Draw a new graph that represents all seven locations, including the new one in Japan. Research the cost of a flight from Seattle (Seattle-Tacoma International Airport) to Kyoto (Kansai International Airport) and add this one new vertex and one new weighted edge to your previous graph. Is this a complete graph?

Adding one new vertex (Kyoto/KIX) and one new edge (WA ↔ Kyoto). Based on current 2026 flight data, a one-way ticket from Seattle-Tacoma (SEA) to Kansai International (KIX) costs approximately $800.



A complete graph K_7 is a graph where there is an edge (connection) between every two vertices (nodes), which means that to complete K_7, there are 21 edges that are required; using the formula, 21 edges are determined by $n(n-1)/2 \Rightarrow 7(6)/2 = 21$. This chart has only 16 edges; the initial 15 from K_6 and the one new edge between Seattle and Kyoto. However, the remaining 5 edges that would connect Kyoto to Dallas, Atlanta, Newark, Buffalo, and Chicago do not exist. Thus, K_7 is not complete, because K_7 is not a complete graph.

5. With this new international location added, it is possible to continue to add

weighted edges to create a complete graph containing seven vertices. In

graph theory, we call this the K7 graph. How many unique travel routes can

be found to travel to all seven of these locations once and return to Seattle?

That is, find the number of unique Hamilton circuits for the complete graph.

K7.

The total number of Hamilton cycles through n vertices on a complete graph can be calculated with the following formula:

$1/2(n-1)!$

So for K7 (where n=7):

$1/2(7-1)! = 1/2(6!) = 720/2 = 360$

So there are a total of 360 unique Hamilton circuits on K7!

6! permutations of 6 cities:

Rationale: I will fix Seattle as the point where the Hamilton circuit starts and ends. There are 6 other cities. The remaining cities can be arranged in 720 (6!) different ways. Every Hamilton circuit can be traveled to (in clockwise direction) and traveled to the same city in the opposite direction (in counterclockwise direction), which means that each Hamilton circuit has 2 different ways to travel the same city. Therefore, I divide the total number of Hamilton circuits by 2 to find the number of unique Hamilton circuits on K7; thus, there are 360 unique circuits. To verify the 360 unique Hamilton circuits, it would be necessary to use brute force to check each of them; this is the reason that the traveling salesman problem is computationally difficult!